Realization:

↳ to realize any function, this function must be PRF

There are 5 methods of Realization:

1) Elementary
2) Foster I ⟶ For Reactance [Z(S)]
3) Foster II ⟶ For Susceptance [Y(S)]
4) Cour I
5) Cour II

Basic information:

if

$$Z(S) = \underbrace{Z_1(S) + Z_2(S)}$$

This means that $Z_1(S)$ series with $Z_2(S)$

if

$$Y(S) = \underbrace{Y_1(S) + Y_2(S)}$$

This means that $Y_1(S)$ parallel with $Y_2(S)$

if

$Z(S) = \square + \frac{1}{\square}$, $Y(S) = \square + \frac{1}{\square}$

$Z(S)$; $Y_2(S)$; $Y_1(S)$; $Z_2(S)$

1) Elementary Method: .. ( ) [2]

"we are trying to separate the equation"

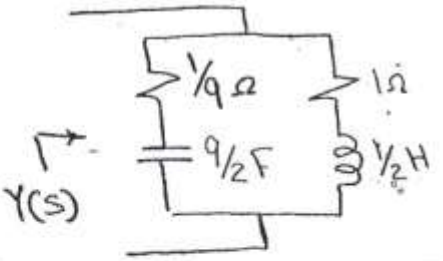
ex:

$$Y(S) = \frac{9S+2}{S+2}$$

$$\therefore Y(S) = \frac{9S}{S+2} + \frac{2}{S+2} = \frac{1}{\frac{1}{9}+\frac{2}{9S}} + \frac{1}{\frac{S}{2}+1}$$

$$Y(S) = \frac{1}{\frac{1}{9}+\frac{1}{9/2S}} + \frac{1}{S/2+1}$$

Z Z Z Z



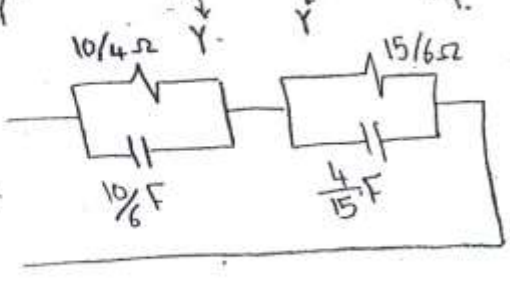
ex:

$$Z(S) = \frac{10S+15}{4S+6}$$

$$\therefore Z(S) = \frac{10S}{4S+6} + \frac{15}{4S+6} = \frac{1}{\frac{4}{10}+\frac{6}{10S}} + \frac{1}{\frac{4}{15}S+\frac{6}{15}}$$

$$\therefore Z(S) = \frac{1}{\frac{4}{10}+\frac{1}{10/6S}} + \frac{1}{\frac{4}{15}S+\frac{6}{15}}$$

Y Y Y Y



ex:

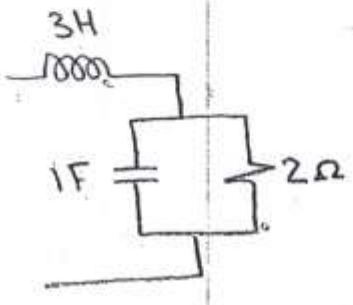
$$Z(S) = \frac{6S^2+3S+2}{2S+1}$$

$$\therefore Z(S) = \frac{3S(2S+1)+2}{2S+1} = \frac{3S(2S+1)}{2S+1} + \frac{2}{2S+1}$$

$$\therefore Z(S) = 3S + \frac{1}{S+1/2}$$

Z

Y Y



ex:

$$Z(S) = \frac{S^2+2S}{S+4}$$

∵ Degree of $N(S) > D(S)$ ∴ we will make long Division

$$\begin{array}{r} S \\ S+4 \overline{)\,S^2+2S} \\ \underline{S^2+4S} \\ -2S \end{array}$$

$$\therefore Z(S) = S - \frac{2S}{S+4}$$

$$\therefore Z(S) = S - \frac{1}{\frac{1}{2}+\frac{4}{2S}} = S \ominus \frac{1}{\frac{1}{2}+\frac{1}{\frac{1}{2}S}}$$

there is a problem due to −ve sign.

Check PRF

$m_1 = S^2$, $n_1 = 2S$

$m_2 = 4$, $n_2 = S$

$$\therefore m_1 m_2 - n_1 n_2 \Big|_{S=j\omega} \geq 0 \ \forall \omega \Rightarrow 4S^2 - 2S^2 \Big|_{S=j\omega} \geq 0 \ \forall \omega$$

$$2S^2 \Big|_{S=j\omega} \geq 0 \ \forall \omega, \quad -2\omega^2 \not\geq 0 \ \forall \omega$$

∴ not PRF ∴ Can't be realized..

ex:

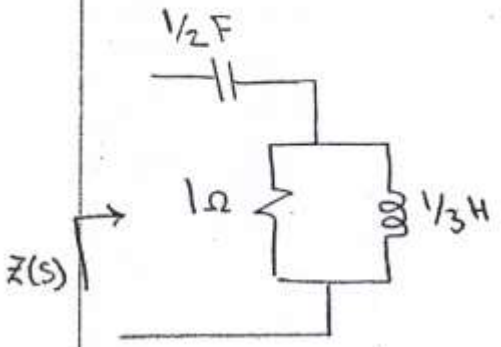
$$Z(S) = \frac{S^2+2S+6}{S(S+3)} = \frac{2(S+3)+S^2}{S(S+3)} = \frac{2(S+3)}{S(S+3)} + \frac{S^2}{S(S+3)}$$

$$Z(S) = \frac{2}{S} + \frac{1}{\frac{1}{S}(S+3)}$$

$$= \frac{2}{S} + \frac{1}{1+\frac{3}{S}}$$

$$Z(S) = \frac{1}{\frac{1}{2}S} + \frac{1}{1+\frac{1}{\frac{1}{3}S}}$$

y y y



Network

LC

$Z(S) = \frac{N(S)}{D(S)}$ or $Y(S) = \frac{N(S)}{D(S)}$

* $\frac{N(S)}{D(S)} = \frac{\text{even}}{\text{odd}}$ or $\frac{\text{odd}}{\text{even}}$

Power of s in num. & den. $\frac{\text{odd}}{\text{even}}$ or $\frac{\text{even}}{\text{odd}}$ *

e.g. $\frac{S^2+1}{S} \Rightarrow \frac{\text{even}}{\text{odd}}$

or $\frac{S^2+S}{S^2+1} \Rightarrow \frac{\text{odd}}{\text{even}}$

RC

$Z(S) = \frac{N(S)}{D(S)}$ or $Y(S) = \frac{N(S)}{D(S)}$

* $\frac{N(S)}{D(S)} = \frac{\text{even+odd}}{\text{even+odd}}$

e.g. $\frac{S^2+S+1}{S+2} \Rightarrow \frac{\text{even+odd}}{\text{even+odd}}$

* $Z(0) \geq Z(\infty)$
$Y(0) \leq Y(\infty)$

RL

$Z(S) = \frac{N(S)}{D(S)}$ or $Y(S) = \frac{N(S)}{D(S)}$

* $\frac{N(S)}{D(S)} = \frac{\text{even+odd}}{\text{even+odd}}$

* $Z(0) \leq Z(\infty)$
$Y(0) \geq Y(\infty)$

* Foster I "$Z(S) = \ldots$"

$$Z_{LC}(S) = sK_\infty + \frac{K_0}{S} + \sum_{i=1}^{n} \frac{2K_i S}{S^2+\omega_i^2}$$

$$K_\infty = \lim_{S\to\infty} \frac{Z(S)}{S}$$

$$K_0 = \lim_{S\to 0} SZ(S)$$

$$2K_i = \lim_{S^2\to -\omega_i^2} \frac{Z(S)(S^2+\omega_i^2)}{S}$$

$\Rightarrow \frac{\text{odd}}{\text{even}}$ or $\frac{\text{even}}{\text{odd}}$

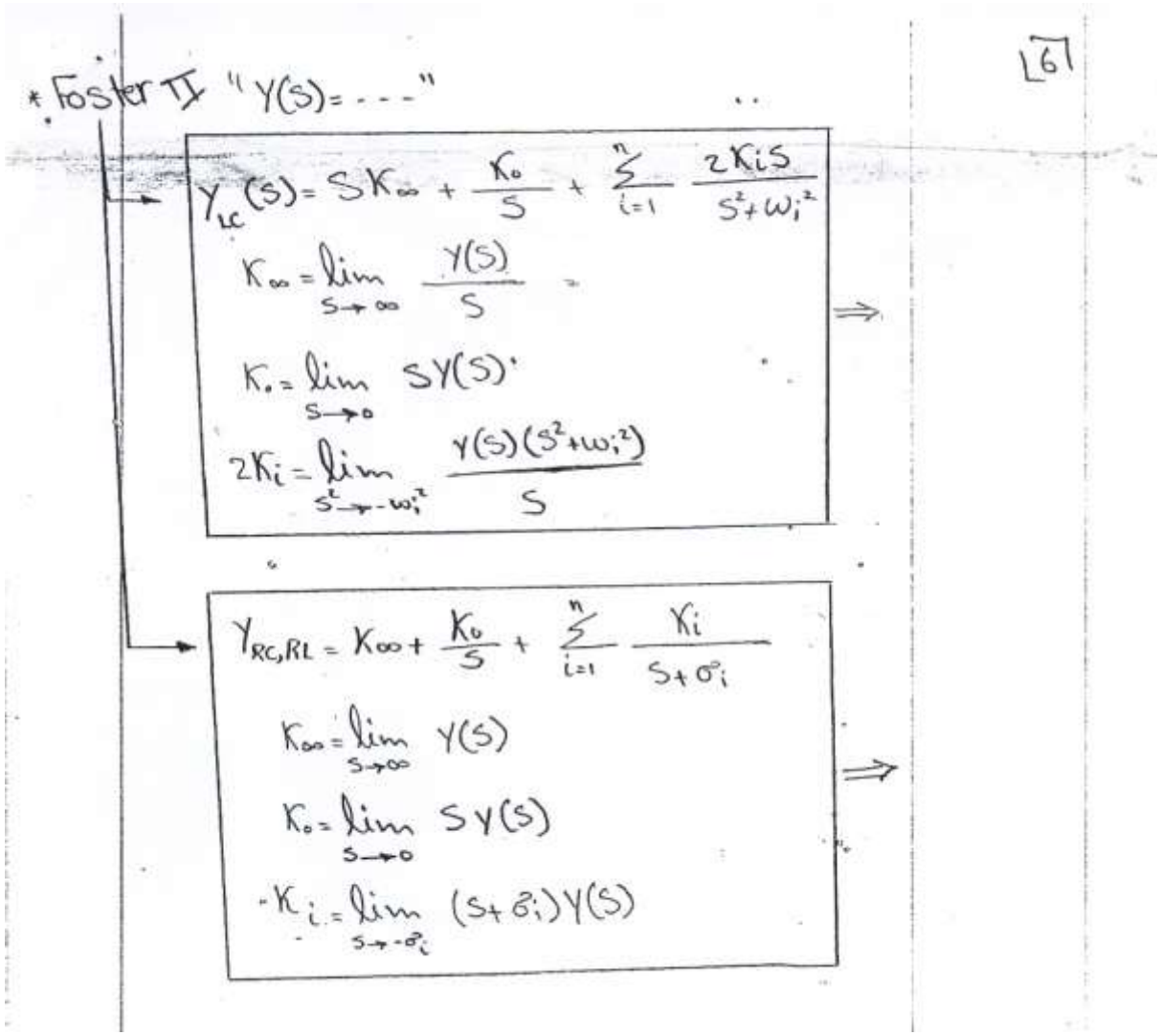
$$Z_{RC,RL} = K_\infty + \frac{K_0}{S} + \sum_{i=1}^{n} \frac{K_i}{S+\sigma_i}$$

$$K_\infty = \lim_{S\to\infty} Z(S)$$

$$K_0 = \lim_{S\to 0} SZ(S)$$

$$K_i = \lim_{S\to -\sigma_i} (S+\sigma_i)Z(S)$$

$\Rightarrow \frac{\text{even+odd}}{\text{even+odd}}$

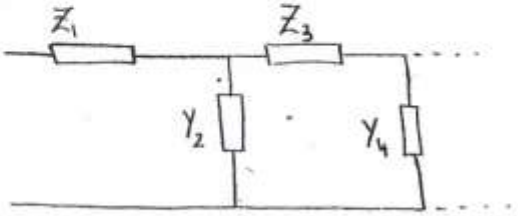

* Foster II "Y(S)= - - -"
$Y_{LC}(S) = SK_\infty + \frac{K_0}{S} + \sum_{i=1}^{n} \frac{2K_i S}{S^2+\omega_i^2}$
$K_\infty = \lim_{S\to\infty} \frac{Y(S)}{S}$
$K_0 = \lim_{S\to 0} SY(S)$
$2K_i = \lim_{S^2\to -\omega_i^2} \frac{Y(S)(S^2+\omega_i^2)}{S}$
$Y_{RC,RL} = K_\infty + \frac{K_0}{S} + \sum_{i=1}^{n} \frac{K_i}{S+\sigma_i}$
$K_\infty = \lim_{S\to\infty} Y(S)$
$K_0 = \lim_{S\to 0} SY(S)$
$K_i = \lim_{S\to -\sigma_i} (S+\sigma_i)Y(S)$

* Cauer I "$Z(S) = \cdots$ or $Y(S) = \cdots$"

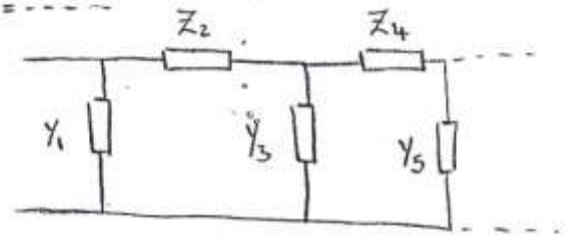
→ $\lim_{S \to \infty} Z(S) = \infty$ or $\lim_{S \to \infty} Y(S) = \infty$

→ Rearrange $N(S)$ & $D(S)$ in descending order $\left(\frac{S^4+S^3+S^2+S+1}{S^3+S^2+S+1}\right)$

i) if $Z(S) = \cdots$



ii) if $Y(S) = \cdots$

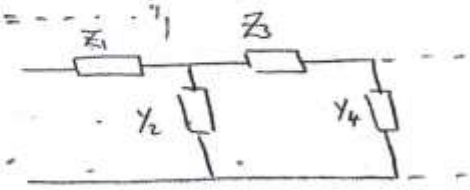


* Cauer II "$Z(S) = \cdots$ or $Y(S) = \cdots$"

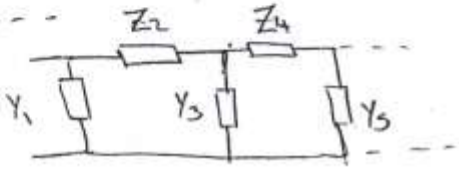
→ $\lim_{S \to 0} Z(S) = \infty$ or $\lim_{S \to 0} Y(S) = \infty$

→ Rearrange $N(S)$ & $D(S)$ in ascending order $\left(\frac{1+S+S^2+\cdots}{1+S+S^2+\cdots}\right)$

i) if $Z(S) = \cdots$



ii) if $Y(S) = \cdots$



Realize using Foster I

$$Z(S) = \frac{2S^4 + 20S^2 + 18}{S^3 + 4S} = \frac{2(S^4 + 10S^2 + 9)}{S(S^2 + 4)}$$

$\because \frac{\text{even}}{\text{odd}} \therefore$ LC Network

$$Z_{LC}(S) = SK_\infty + \frac{K_0}{S} + \frac{2K_1 S}{S^2 + 4}$$

$$K_\infty = \lim_{S \to \infty} \frac{Z(S)}{S} = \lim_{S \to \infty} \frac{2(S^4 + 10S^2 + 9)}{S^2(S^2 + 4)} = 2$$

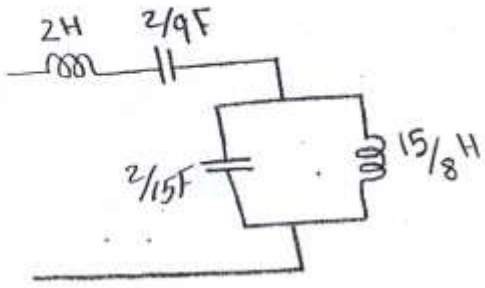
$$K_0 = \lim_{S \to 0} SZ(S) = \lim_{S \to 0} \frac{2(S^4 + 10S^2 + 9)}{S^2 + 4} = 9/2$$

$$2K_1 = \lim_{S^2 \to -4} \frac{(S^2+4)Z(S)}{S} = \lim_{S^2 \to -4} \frac{2(S^4 + 10S^2 + 9)}{S^2} = \frac{2(16 - 10\times4 + 9)}{-4} = \frac{15}{2}$$

$$\therefore Z_{LC}(S) = 2S + \frac{9/2}{S} + \frac{15/2\,S}{S^2 + 4}$$

$$Z_{LC}(S) = 2S + \frac{1}{2/9S} + \frac{1}{\frac{2}{15}S + \frac{8}{15S}}$$

$$Z_{LC}(S) = 2S + \frac{1}{2/9S} + \frac{1}{2/15S + \frac{1}{15/8\,S}}$$



* $\lim_{x \to \infty} K \cdot \frac{N(S)}{D(S)} = 0$ if deg. of D(S) > deg. of N(S)

* $\lim_{x \to \infty} K \cdot \frac{N(S)}{D(S)} = \infty$ if deg. of D(S) < deg. of N(S)

* $\lim_{x \to \infty} K \cdot \frac{N(S)}{D(S)} = K$ if deg. of D(S) = deg. of N(S)

$$Z(S) = \frac{2S(S^2+4)(S^2+16)}{(S^2+1)(S^2+9)}$$

odd / even ∴ LC Network

$$Z_{LC}(S) = SK_\infty + \frac{K_0}{S} + \frac{2K_1 S}{S^2+1} + \frac{2K_2 S}{S^2+9}$$

$$K_\infty = \lim_{S\to\infty} \frac{Z(S)}{S} = \lim_{S\to\infty} \frac{2(S^2+4)(S^2+16)}{(S^2+1)(S^2+9)} = 2$$

$$K_0 = \lim_{S\to 0} SZ(S) = \lim_{S\to 0} \frac{2S^2(S^2+4)(S^2+16)}{(S^2+1)(S^2+9)} = 0$$

$$2K_1 = \lim_{S^2\to -1} \frac{(S^2+1)Z(S)}{S} = \lim_{S^2\to -1} \frac{2(S^2+4)(S^2+16)}{S^2+9} = \frac{2(-1+4)(-1+16)}{-1+9} = \frac{45}{4}$$

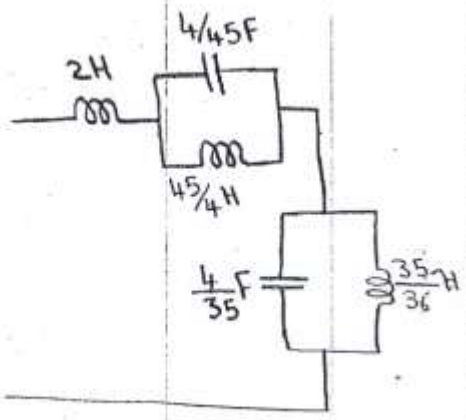
$$2K_2 = \lim_{S^2\to -9} \frac{(S^2+9)Z(S)}{S} = \lim_{S^2\to -9} \frac{2(S^2+4)(S^2+16)}{S^2+1} = \frac{2(-9+4)(-9+16)}{-9+1} = \frac{35}{4}$$

$$Z_{LC}(S) = 2S + 0 + \frac{45/4\,S}{S^2+1} + \frac{35/4\,S}{S^2+9}$$

$$= 2S + \frac{1}{\frac{4}{45}S + \frac{4}{45S}} + \frac{1}{\frac{4}{35}S + \frac{36}{35S}}$$

$$Z_{LC}(S) = 2S + \frac{1}{\frac{4}{45}S + \frac{1}{\frac{45}{4}S}} + \frac{1}{\frac{4}{35}S + \frac{1}{\frac{35}{36}S}}$$

Z → Y → Y → Y → Y



Synthesis using Foster I

$$Z(S) = \frac{(S+1)(S+3)}{S(S+2)}$$

$\because \frac{\text{even+odd}}{\text{even+odd}}$ $\therefore$ it may be RL or RC Network

$Z(0) = \infty$, $Z(\infty) = 1$

$\because Z(0) > Z(\infty)$ $\therefore$ it is RC Network

$$Z_{RC}(S) = K_\infty + \frac{K_0}{S} + \frac{K_1}{S+2}$$

$$K_\infty = \lim_{S\to\infty} Z(S) = \lim_{S\to\infty} \frac{(S+1)(S+3)}{S(S+2)} = 1$$

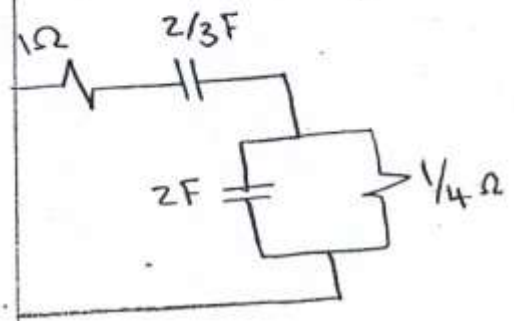
$$K_0 = \lim_{S\to 0} SZ(S) = \lim_{S\to 0} \frac{(S+1)(S+3)}{S+2} = 3/2$$

$$K_1 = \lim_{S\to -2} (S+2)Z(S) = \lim_{S\to -2} \frac{(S+1)(S+3)}{S} = \frac{(-2+1)(-2+3)}{-2} = 1/2$$

$$Z_{RC}(S) = 1 + \frac{3/2}{S} + \frac{1/2}{S+2}$$

$$Z_{RC}(S) = 1 + \frac{1}{2/3\,S} + \frac{1}{2S+4}$$

Z Y Y Y



Realize using Foster I

$$Z(S) = \frac{2(S+1)(S+3)}{(S+2)(S+6)}$$

$\frac{\text{even}+\text{odd}}{\text{even}+\text{odd}}$ ∴ it may be RL or RC Network

$Z(0) = 1/2$, $Z(\infty) = 2$

$Z(0) < Z(\infty)$ ∴ it is RL Network

$$Z_{RL}(S) = K_\infty + \frac{K_0}{S} + \frac{K_1}{S+2} + \frac{K_2}{S+6}$$

$$K_\infty = \lim_{S\to\infty} Z(S) = \lim_{S\to\infty} \frac{2(S+1)(S+3)}{(S+2)(S+6)} = 2$$

$$K_0 = \lim_{S\to 0} SZ(S) = \lim_{S\to 0} \frac{2S(S+1)(S+3)}{(S+2)(S+6)} = 0$$

$$K_1 = \lim_{S\to -2} (S+2)Z(S) = \lim_{S\to -2} \frac{2(S+1)(S+3)}{(S+6)} = \frac{2(-2+1)(-2+3)}{-2+6} = -1/2$$

لو وجدت -ve لازم تفرض $Z(S)$

$$Z'(S) = \frac{Z(S)}{S}$$

$$Z'(S) = \frac{2(S+1)(S+3)}{S(S+2)(S+6)}$$

$$Z'(S) = \frac{2(S+1)(S+3)}{S(S+2)(S+6)}$$

$$K_\infty = \lim_{S\to\infty} Z'(S) = \lim_{S\to\infty} \frac{2(S+1)(S+3)}{S(S+2)(S+6)} = 0$$

$$K_0 = \lim_{S\to 0} SZ'(S) = \lim_{S\to 0} \frac{2(S+1)(S+3)}{(S+2)(S+6)} = 1/2$$

$$K_1 = \lim_{S\to -2} (S+2)Z'(S) = \lim_{S\to -2} \frac{2(S+1)(S+3)}{S(S+6)} = \frac{2(-2+1)(-2+3)}{-2(-2+6)} = 1/4$$

$$K_2 = \lim_{S\to -6} (S+6)Z'(S) = \lim_{S\to -6} \frac{2(S+1)(S+3)}{S(S+2)} = \frac{2(-6+1)(-6+3)}{-6(-6+2)} = 5/4$$

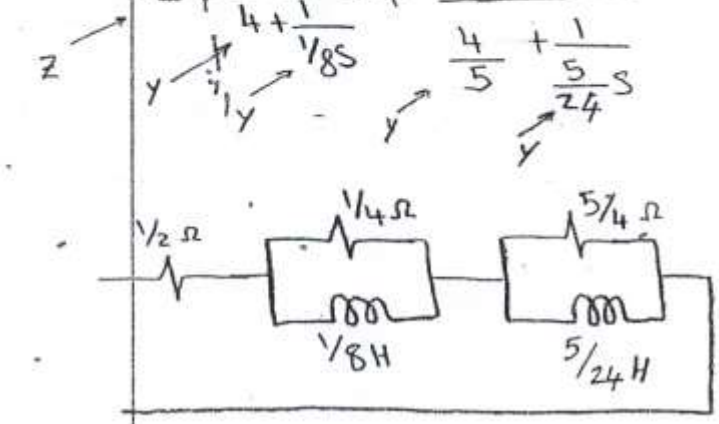
$$Z'(S) = 0 + \frac{1/2}{S} + \frac{1/4}{S+2} + \frac{5/4}{S+6}$$

$$\because Z'(S) = \frac{Z(S)}{S} \quad \therefore Z(S) = SZ'(S)$$

$$Z(S) = 1/2 + \frac{1/4\,S}{S+2} + \frac{5/4\,S}{S+6}$$

$$= 1/2 + \frac{1}{4+\frac{8}{S}} + \frac{1}{\frac{4}{5}+\frac{24}{5S}} =$$

$$Z(S) = 1/2 + \frac{1}{4+\frac{1}{1/8S}} + \frac{1}{\frac{4}{5}+\frac{1}{\frac{5}{24}S}}$$



Realize using Foster II

$$y(s) = \frac{(s^2+1)(s^2+9)}{2s(s^2+4)(s^2+16)}$$

$\frac{\text{even}}{\text{odd}}$ ∴ LC Network

$$y(s) = sK_\infty + \frac{K_0}{s} + \frac{2K_1 s}{s^2+4} + \frac{2K_2 s}{s^2+16}$$

$$K_\infty = \lim_{s\to\infty} \frac{y(s)}{s} = \lim_{s\to\infty} \frac{(s^2+1)(s^2+9)}{2s^2(s^2+4)(s^2+16)} = 0$$

$$K_0 = \lim_{s\to 0} s\,y(s) = \lim_{s\to 0} \frac{(s^2+1)(s^2+9)}{2(s^2+4)(s^2+16)} = \frac{9}{128}$$

$$2K_1 = \lim_{s^2\to -4} \frac{(s^2+4)y(s)}{s} = \lim_{s^2\to -4} \frac{(s^2+1)(s^2+9)}{2s^2(s^2+16)} = \frac{(-4+1)(-4+9)}{2\times(-4)(-4+16)} = \frac{5}{32}$$

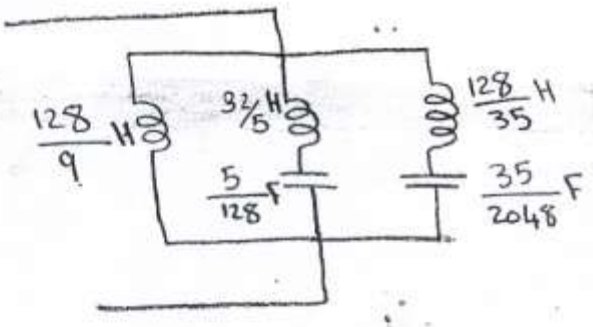
$$2K_2 = \lim_{s^2\to -16} \frac{(s^2+16)y(s)}{s} = \lim_{s^2\to -16} \frac{(s^2+1)(s^2+9)}{2s^2(s^2+4)} = \frac{(-16+1)(-16+9)}{2\times(-16)(-16+4)} = \frac{35}{128}$$

$$y(s) = 0 + \frac{9/128}{s} + \frac{5/32\,s}{s^2+4} + \frac{35/128\,s}{s^2+16}$$

$$= \frac{1}{\frac{128}{9}s} + \frac{1}{\frac{32}{5}s + \frac{128}{5s}} + \frac{1}{\frac{128}{35}s + \frac{2048}{35s}}$$

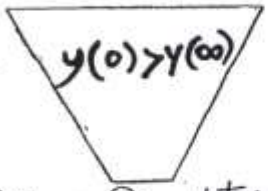
$$y(s) = \frac{1}{\frac{128}{9}s} + \frac{1}{\frac{32}{5}s + \frac{1}{\frac{5}{128}s}} + \frac{1}{\frac{128}{35}s + \frac{1}{\frac{35}{2048}s}}$$

Z Z Z Z Z Z

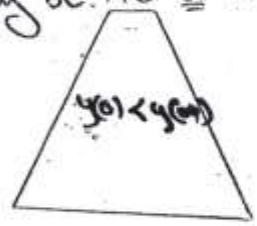


Realize using Foster II

$$y(s) = \frac{s(s+2)}{(s+1)(s+3)}$$



$\frac{\text{even}+\text{odd}}{\text{even}+\text{odd}}$ ∴ it may be RC or RL Network



$y(0) = 0$, $y(\infty) = 1$

∴ $y(0) < y(\infty)$ ∴ it is RC Network

$$y_{RC}(s) = K_\infty + \frac{K_0}{s} + \frac{K_1}{s+1} + \frac{K_2}{s+3}$$

$$K_\infty = \lim_{s\to\infty} y(s) = \lim_{s\to\infty} \frac{s(s+2)}{(s+1)(s+3)} = 1$$

$$K_0 = \lim_{s\to 0} sy(s) = \lim_{s\to 0} \frac{s^2(s+2)}{(s+1)(s+3)} = 0$$

$$K_1 = \lim_{s\to -1} (s+1)Z(s) = \lim_{s\to -1} \frac{s(s+2)}{s+3} = \frac{-1(-1+2)}{1+3} = -1/2$$

$$y'(s) = \frac{y(s)}{s}$$

$$y'(s) = \frac{s+2}{(s+1)(s+3)}$$

$$K_\infty = \lim_{s\to\infty} y'(s) = \lim_{s\to\infty} \frac{s+2}{(s+1)(s+3)} = 0$$

$$K_0 = \lim_{s\to 0} s\,y'(s) = \lim_{s\to 0} \frac{s(s+2)}{(s+1)(s+3)} = 0$$

$$K_1 = \lim_{s\to -1} (s+1)y'(s) = \lim_{s\to -1} \frac{s+2}{s+3} = \frac{-1+2}{-1+3} = \frac{1}{2}$$

$$K_2 = \lim_{s\to -3} (s+3)y'(s) = \lim_{s\to -3} \frac{s+2}{s+1} = \frac{-3+2}{-3+1} = \frac{1}{2}$$

$$y'(s) = 0 + 0 + \frac{1/2}{s+1} + \frac{1/2}{s+3}$$

$$\therefore y'(s) = \frac{y(s)}{s} \quad \therefore y(s) = s\,y'(s)$$

$$\therefore y(s) = \frac{1/2\,s}{s+1} + \frac{1/2\,s}{s+3}$$

$$= \frac{1}{2+\frac{2}{s}} + \frac{1}{2+\frac{6}{s}}$$

$$y(s) = \frac{1}{2+\frac{1}{1/2\,s}} + \frac{1}{2+\frac{1}{1/6\,s}}$$

$2\,\Omega$ $\quad$ $1/2\,F$

$2\,\Omega$ $\quad$ $1/6\,F$

Realize using Ladder Form (May be Cauer I or Cauer II) (1+1)

$$Z(S) = \frac{2S^5 + 12S^3 + 16S}{S^4 + 4S^2 + 3}$$

$\therefore \frac{\text{odd}}{\text{even}}$ $\therefore$ LC Network

$\because \lim_{S \to \infty} Z(S) = \infty \therefore$ Realize be Cauer I

L → Z_1 → $2S$

C → Y_2 → $\frac{S}{4}$

L → Z_3 → $\frac{8S}{3}$

C → Y_4 → $\frac{3}{4}S$

L → Z_5 → $\frac{2}{3}S$

$S^4 + 4S^2 + 3 \;|\; 2S^5 + 12S^3 + 16S$

$2S^5 + 8S^3 + 6S$

$4S^3 + 10S \;|\; S^4 + 4S^2 + 3$

$S^4 + \frac{10}{4}S^2$

$\frac{3}{2}S^2 + 3 \;|\; 4S^3 + 10S$

$4S^3 + 8S$

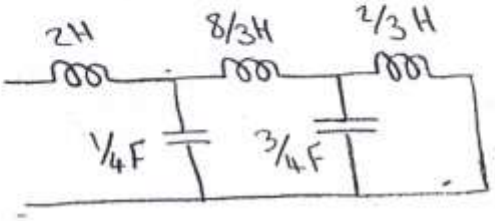
$2S \;|\; \frac{3}{2}S^2 + 3$

$\frac{3}{2}S^2$

$3 \;|\; 2S$

$2S$

0



Synthesis using Cauer I

$$Z(S) = \frac{3(S+2)(S+4)}{S(S+3)}$$

$$Z(S) = \frac{3S^2+18S+24}{S^2+3S}$$

$\because \frac{\text{even+odd}}{\text{even+odd}}$ $\therefore$ it may be RC or RL

$Z(0) = \infty$, $Z(\infty) = 3$

$\because Z(0) > Z(\infty)$ $\therefore$ it is RC Network

$R \rightarrow Z_1 \rightarrow \boxed{3}$

$C \rightarrow y_2 \rightarrow \boxed{\frac{S}{9}}$

$R \rightarrow Z_3 \rightarrow \boxed{27}$

$C \rightarrow y_4 \rightarrow \boxed{\frac{S}{72}}$

$S^2+3S \;\big|\; 3S^2+18S+24$

$3S^2+9S$

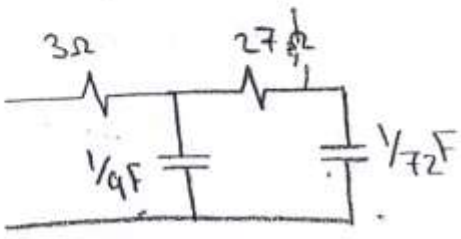
$9S+24 \;\big|\; S^2+3S$

$S^2+8/3S$

$1/3S \;\big|\; 9S+24$

$9S$

$24 \;\big|\; 1/3S$



Realize using Ladder Network:

ex:

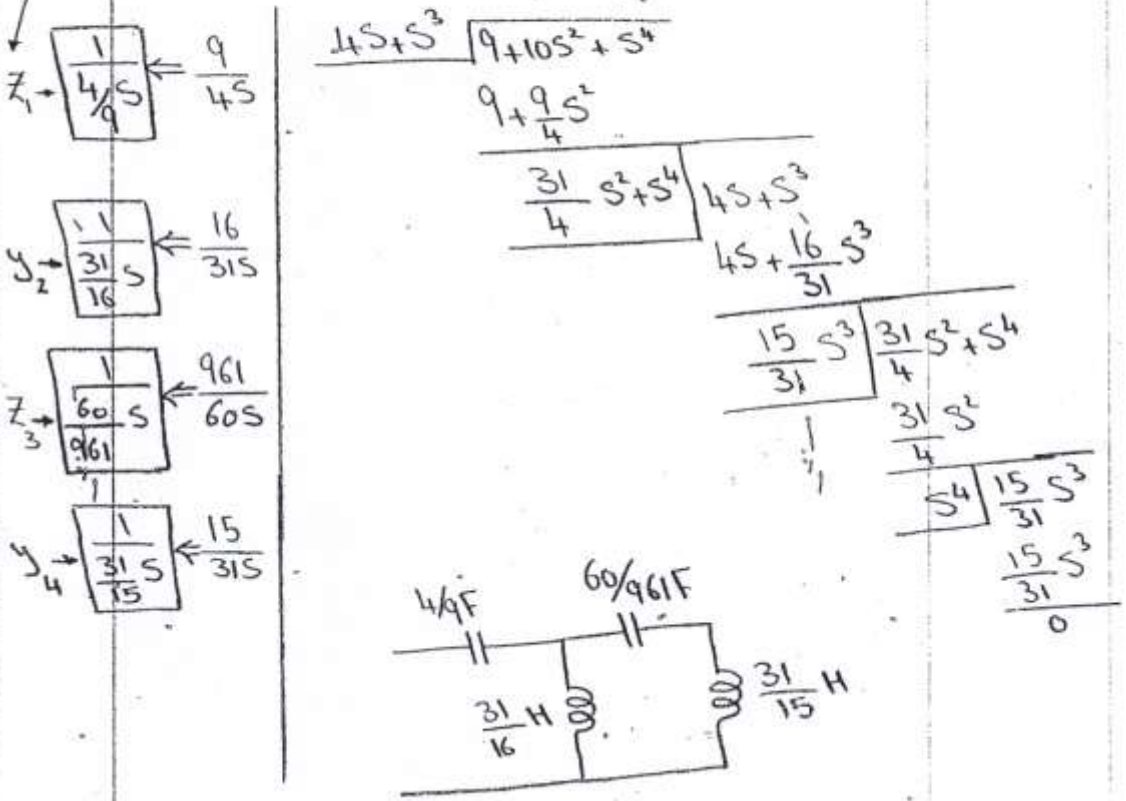
$$Z(S) = \frac{S^4 + 10S^2 + 9}{S^3 + 4S}$$

even / odd $\therefore$ LC Network

$\lim_{S \to 0} Z(S) = \infty$ $\therefore$ Realize using Cauer II

Rearrange Ascendingly

$$Z(S) = \frac{9 + 10S^2 + S^4}{4S + S^3}$$



Synthesis using Cauer II

$$y(s) = \frac{3s^2+18s+24}{s^2+3s}$$

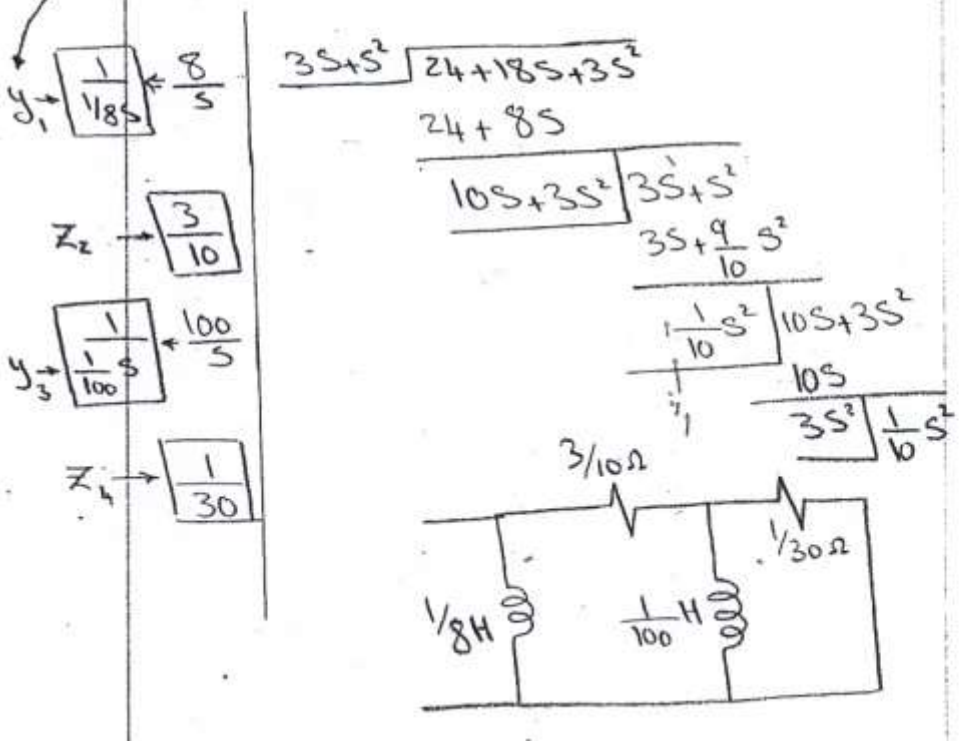
even+odd / even+odd ∴ it may be RC or RL

$y(0) = \infty$, $y(\infty) = 3$

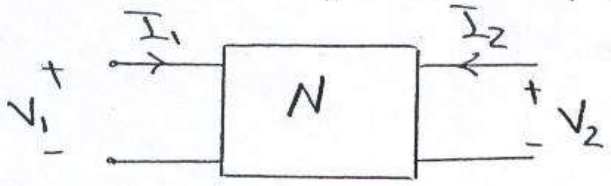
$\therefore y(0) > y(\infty)$ ∴ it is RL Network

Rearrange Ascendingly

$$y(s) = \frac{24+18s+3s^2}{3s+s^2}$$



• Description of two port network



1) $\underset{\text{like impedance}}{Z}$ parameters (open circuit parameters)

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

↓

Z matrix

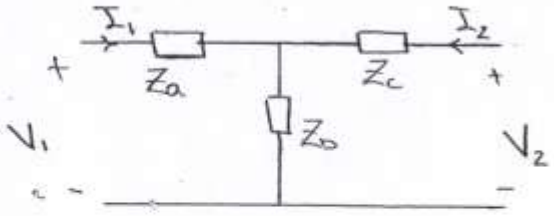
Z_{11}, Z_{22} open ct. driving point impedance.

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \quad , \quad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

Z_{12}, Z_{21} open ct. Transfere impedence

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \quad , \quad Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

ex: Find the Z parameter of the T network



$V_1 = I_1 Z_a + (I_1 + I_2) Z_b$ ---①

$V_2 = I_2 Z_c + (I_1 + I_2) Z_b$ ---②

in [1]

$I_2 = 0$

$\therefore V_1 = I_1 (Z_a + Z_b)$

$\therefore Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = Z_a + Z_b$

$I_1 = 0$

$V_1 = I_2 Z_b$

$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = Z_b$

in [2]

$I_2 = 0$

$V_2 = I_1 Z_b$

$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = Z_b$

$I_1 = 0$

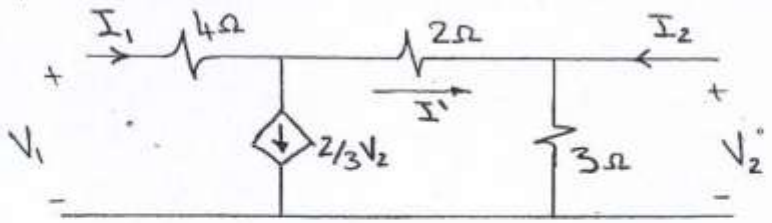
$V_2 = I_2 (Z_c + Z_b)$

$\therefore Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = Z_b + Z_c$

*

[4]

Find the Z parameters of the 2 port network



$$V_1 = 4I_1 + 2I' + 3(I' + I_2) \longrightarrow [1]$$

$$V_2 = 3(I' + I_2) \longrightarrow [2]$$

$$I' = I_1 - \frac{2}{3}V_2$$

in [2] $\Rightarrow V_2 = 3I_1 - 3 \times \frac{2}{3}V_2 + 3I_2$

$$\therefore V_2 = I_1 + I_2$$

in [1]

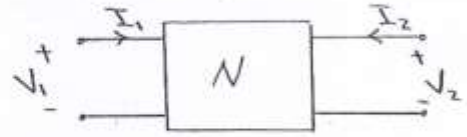
$$V_1 = 4I_1 + 2\left(I_1 - \frac{2}{3}I_1 - \frac{2}{3}I_2\right) + 3\left(I_1 - \frac{2}{3}I_1 - \frac{2}{3}I_2 + I_2\right)$$

$$V_1 = \frac{17}{3}I_1 - \frac{1}{3}I_2$$

$$Z = \begin{pmatrix} 17/3 & -1/3 \\ 1 & 1 \end{pmatrix}$$

The Y parameters (Short Circuit):

↓ like admittance



$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

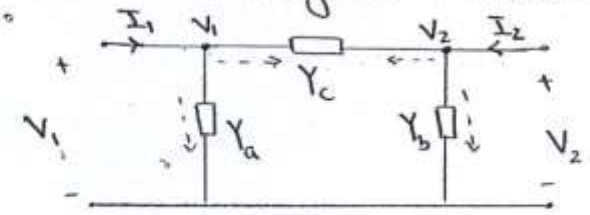
y_{11}, y_{22} Driving point admittance.

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \quad , \quad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

y_{12}, y_{21} Transfer point admittance.

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \quad , \quad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

Ex: Find the Y parameters of the π Model



$$I_1 = V_1 Y_a + (V_1 - V_2)Y_c = V_1(Y_a + Y_c) - V_2 Y_c \longrightarrow [1]$$

$$I_2 = V_2 Y_b + (V_2 - V_1)Y_c = -V_1 Y_c + (Y_b + Y_c)V_2 \longrightarrow [2]$$

in [1]

$V_2 = 0$

$$I_1 = V_1(Y_a + Y_c)$$

$$y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} = Y_a + Y_c$$

$V_1 = 0$

$$I_1 = -V_2 Y_c$$

$$y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0} = -Y_c$$

in [2]

$V_2 = 0$

$$I_2 = -V_1 Y_c$$

$$y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0} = -Y_c$$

$V_1 = 0$

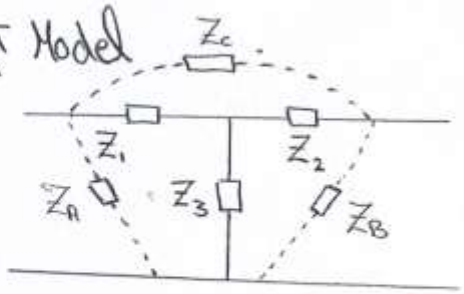
$$I_2 = V_2(Y_b + Y_c)$$

$$y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0} = Y_b + Y_c$$

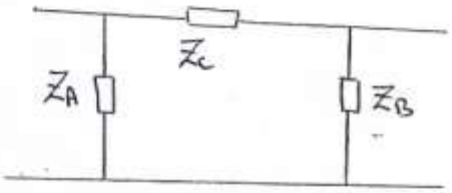
* If $Z_{12} = Z_{21}$ OR $y_{12} = y_{21}$ ∴ Reciprocal system
* If $Z_{11} = Z_{22}$ OR $y_{11} = y_{22}$ ∴ Symmetrical system

Z	Preferable →	T Model
y	Preferable →	π Model

T Model & π Model



T - Model



π (Δ) Model

* Conversion from T to π Model

$$Z_A = \frac{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}{Z_2} \quad \text{(البسط على)}$$

$$, Z_B = \frac{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}{Z_1}$$

$$, Z_C = \frac{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}{Z_3}$$

Conversion from π to T Model

$$Z_1 = \frac{Z_AZ_C}{Z_A + Z_B + Z_C} \quad \text{(بضرب على بعض)}$$

$$, Z_2 = \frac{Z_BZ_C}{Z_A + Z_B + Z_C}$$

$$, Z_3 = \frac{Z_AZ_B}{Z_A + Z_B + Z_C}$$

3] Hybrid "H" parameters
"It Contains both open ct. & short ct."

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

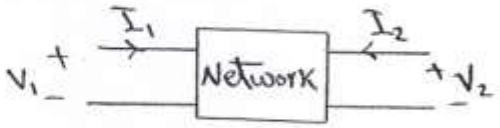
$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$ Short ct. driving point impedance

$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$ open ct. Transfere voltage ratio.

$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$ Short ct. Transfere Current ratio

$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$ open ct. driving point admittance.

1] Transmission parameter "T"



$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} +V_2 \\ -I_2 \end{bmatrix}$$

$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$ open ct. Transfere Voltage ratio.

$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0}$ Short ct. transfere impedance.

$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$ open ct. Transfere admittance.

$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0}$ Short ct. Transfere Current ratio.

Relationship among Two port parameters

1] $Z \longrightarrow Y$

$Y = [Z]^{-1}$

2] $Y \longrightarrow Z$

$Z = [Y]^{-1}$

3] $T \longrightarrow Z$

$Z_{11} = \frac{A}{C}$, $Z_{12} = \frac{AD}{C} - B$

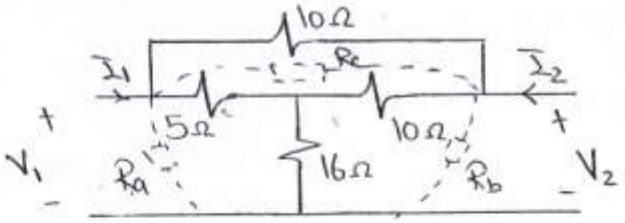
$Z_{21} = \frac{1}{C}$, $Z_{22} = \frac{D}{C}$

4] $Z \longrightarrow T$

$A = \frac{Z_{11}}{Z_{21}}$, $B = \frac{Z_{11} \times Z_{22}}{Z_{21}} - Z_{12}$

$C = \frac{1}{Z_{21}}$, $D = \frac{Z_{22}}{Z_{21}}$

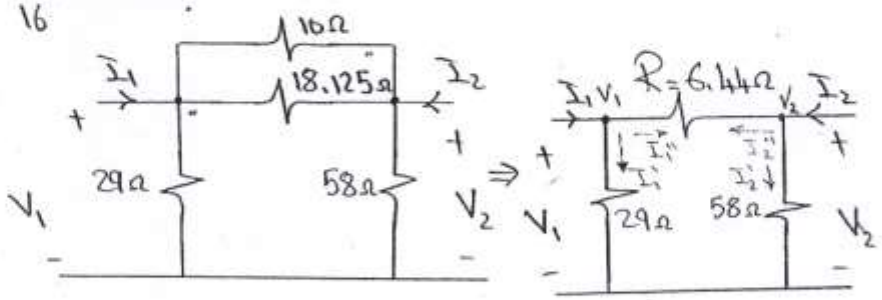
Find H-parameter



Solution

$$R_a = \frac{5\times10+5\times16+16\times10}{10} = 29\Omega, \quad R_b = \frac{5\times10+5\times16+16\times10}{5} = 58\Omega$$

$$R_c = \frac{5\times10+5\times16+16\times10}{16} = 18.125\Omega$$



$$R = (10//18.125) = \frac{10\times18.125}{10+18.125} = 6.44\Omega$$

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$V_1 = 29I'$$
$$I' = I_1 - I_1''$$
$$= I_1 - \frac{V_1 - V_2}{R}$$

[6]

$$\therefore V_1 = 29 I_1 - \frac{29}{R} V_1 + \frac{29}{R} V_2$$

$$\therefore V_1\left(1 + \frac{29}{R}\right) = 29 I_1 + \frac{29}{R} V_2$$

$$\frac{R+29}{R} V_1 = 29 I_1 + \frac{29}{R} V_2$$

$$\therefore V_1 = \frac{29R}{R+29} I_1 + \frac{29}{R+29} V_2$$

$$V_1 = \frac{29 \times 6.44}{6.44+29} I_1 + \frac{29}{6.44+29} V_2 = 5.27 I_1 + 0.82 V_2$$

$$\therefore V_1 = 5.27 I_1 + 0.82 V_2 \longrightarrow [1]$$

$$I_2 = I_2' + I_2''$$

$$= \frac{V_2}{58} + \frac{V_2 - V_1}{6.44} = 0.172 V_2 - \frac{1}{6.44} V_1$$

from [1]

$$\therefore I_2 = 0.172 V_2 - \frac{5.27}{6.44} I_1 - \frac{0.82}{6.44} V_2$$

$$I_2 = -0.82 I_1 + 0.045 V_2 \longrightarrow [2]$$

$$\therefore h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = 5.27$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = 0.82$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = -0.82$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = 0.045$$

$$h = \begin{pmatrix} 5.27 & 0.82 \\ -0.82 & 0.045 \end{pmatrix}$$

18

Find the Transmission parameters of the Networks shown in fig. 1 & fig. 2

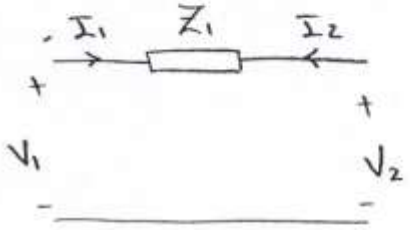


fig (1)

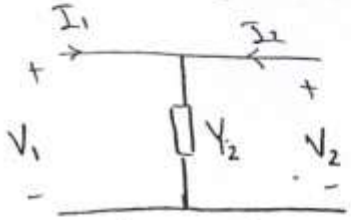


fig (2)

For Fig(1)

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

$\therefore V_1 = -I_2 Z_1 + V_2 \quad \therefore V_1 = V_2 - I_2 Z_1 \longrightarrow [1]$

$I_1 = -I_2 \longrightarrow [2]$

$$\therefore A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = 1 \quad , B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = Z_1 \quad , C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = 0 \quad , D = \left.\frac{I_2}{-I_1}\right|_{V_2=0} = 1$$

$$\therefore T = \begin{pmatrix} 1 & Z_1 \\ 0 & 1 \end{pmatrix}$$

or Fig (2)

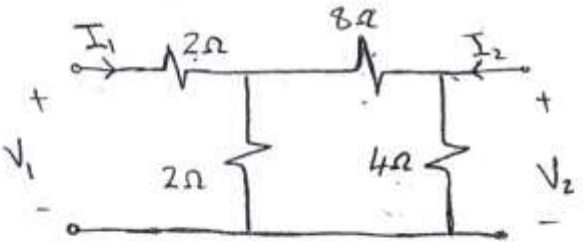
$\therefore V_1 = V_2 \longrightarrow [1]$

$V_2 = \frac{I_1 + I_2}{Y_2} \quad \therefore I_1 = Y_2 V_2 - I_2 \longrightarrow [2]$

$\therefore A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = 1 \;,\; B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = 0 \;,\; C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = Y_2 \;,\; D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = 1$

$\therefore T = \begin{pmatrix} 1 & 0 \\ Y_2 & 1 \end{pmatrix}$

Using the Results above, Find the Transmission parameters of the Network shown in fig. 3



$T = T_1 * T_2 * T_3 * T_4$

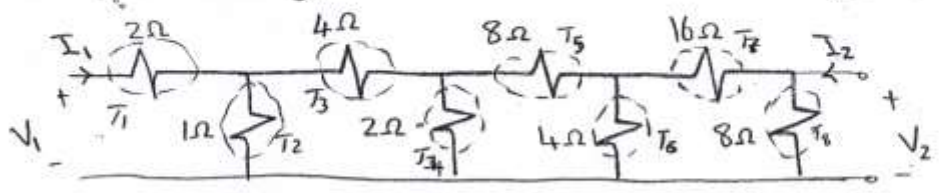
$T_1 = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \;,\; T_2 = \begin{pmatrix} 1 & 0 \\ 1/2 & 1 \end{pmatrix} \;,\; T_3 = \begin{pmatrix} 1 & 8 \\ 0 & 1 \end{pmatrix} \;,\; T_4 = \begin{pmatrix} 1 & 0 \\ 1/4 & 1 \end{pmatrix}$

$$T = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} * \begin{pmatrix} 1 & 0 \\ 1/2 & 1 \end{pmatrix} * \begin{pmatrix} 1 & 8 \\ 0 & 1 \end{pmatrix} * \begin{pmatrix} 1 & 0 \\ 1/4 & 1 \end{pmatrix}$$

$$T = \begin{pmatrix} 2 & 2 \\ 1/2 & 1 \end{pmatrix} \begin{pmatrix} 3 & 8 \\ 1/4 & 1 \end{pmatrix} = \begin{pmatrix} 13/2 & 18 \\ 7/4 & 5 \end{pmatrix}$$

$\therefore A = \frac{13}{2}$, $B = 18$, $C = 7/4$, $D = 5$

Find T. parameters of the 2 port network shown in fig. (4)



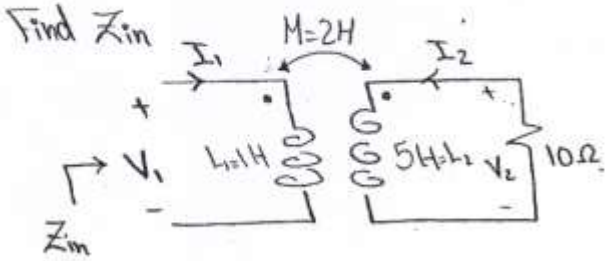
$\therefore T = T_1 * T_2 * T_3 * T_5 * T_6 * T_7 * T_8$

$$T = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1/2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 8 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1/4 & 1 \end{pmatrix} \begin{pmatrix} 1 & 16 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1/8 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 3 & 4 \\ 1/2 & 1 \end{pmatrix} \begin{pmatrix} 3 & 8 \\ 1/4 & 1 \end{pmatrix} \begin{pmatrix} 3 & 16 \\ 1/8 & 1 \end{pmatrix}$$

$$T = \begin{pmatrix} 10 & 14 \\ 7/2 & 5 \end{pmatrix} \begin{pmatrix} 10 & 56 \\ 7/8 & 5 \end{pmatrix} = \begin{pmatrix} \frac{449}{4} & 630 \\ \frac{315}{8} & 221 \end{pmatrix}$$

Find Z_{in}



If dots are Similar : Coupling +ve
If dots are opposite : Coupling -ve.

Solution

$$Z_{in} = \frac{V_1}{I_1}$$

$$V_1 = L_1 \frac{dI_1}{dt} + M \frac{dI_2}{dt}$$

$$V_1(S) = L_1 S I_1(S) - L_1 I_1(0) + M S I_2(S) - M I_2(0)$$

(z.i.c) (z.i.c)

$$\therefore V_1(S) = L_1 S I_1(S) + M S I_2(S) \longrightarrow [1]$$

$$V_2(S) = L_2 S I_2(S) + M S I_1(S) \longrightarrow [2]$$

$$V_2 = -I_2 R_L$$

$$-I_2 R_L = L_2 S I_2 + M S I_1$$

$$M S I_1 = -(R_L + S L_2) I_2 \quad \therefore I_2 = \frac{-M S I_1}{R_L + S L_2}$$

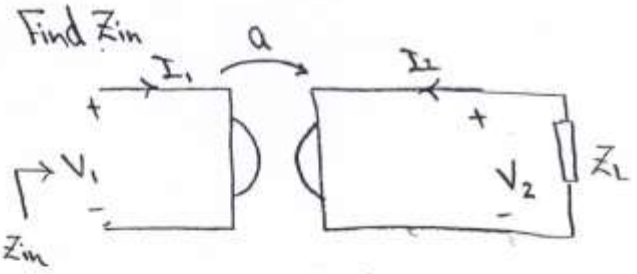
in [2]

$$\therefore V_1 = \left[L_1 S + \frac{M^2 S^2}{R_L + S L_2} \right] I_1$$

$$\frac{V_1}{I_1} = Z_{in} = L_1 S - \frac{(MS)^2}{R_L + S L_2} = 5 - \frac{4S^2}{10 + 5S} = \frac{5S^2 + 10S - 4S^2}{5S + 10}$$

$$\therefore Z_{in} = \frac{S^2 + 10}{5S + 10}$$

Find Z_{in}



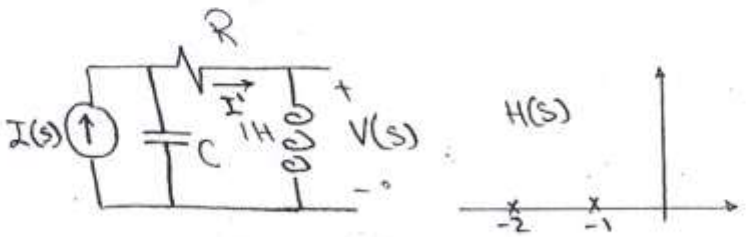
Solution.

$$Z_{in} = \frac{V_1}{I_1}$$

$$V_1 = -a I_2 \longrightarrow [1]$$
$$V_2 = a I_1 \longrightarrow [2]$$

$$V_2 = -I_2 Z_L$$
$$\therefore -I_2 Z_L = a I_1$$
$$\therefore I_2 = \frac{-a}{Z_L} I_1$$

in [1] $\therefore V_1 = \frac{a^2}{Z_L} I_1 \therefore \frac{V_1}{I_1} = Z_{in} = \frac{a^2}{Z_L}$



Find R, C

Solution.

$$H(S) = \frac{V(S)}{I(S)} \quad , \quad V(S) = Z_L I' = SL \times I'$$

$$I' = I \times \frac{1/SC}{R + LS + 1/SC} = I \times \frac{1}{S^2LC + RSC + 1} \quad \therefore V(S) = I \times \frac{SL}{S^2LC + SRC + 1}$$

$$\therefore H(S) = \frac{V(S)}{I(S)} = \frac{SL}{S^2LC + SRC + 1} = \frac{S}{S^2C + SRC + 1}$$

[27]

C/C eqn: $S^2C + SRC + 1 = 0 \longrightarrow [1]$

$\therefore$ has 2 poles at -2 & -1

$\therefore (S+2)(S+1) = 0$

$\therefore S^2 + 3S + 2 = 0 \quad \therefore \frac{1}{2}S^2 + \frac{3}{2}S + 1 = 0 \longrightarrow [2]$

by Comparing [1] & [2]

$\therefore C = \frac{1}{2}F$ #

$RC = \frac{3}{2} \quad \therefore R = \frac{3}{2} \times \frac{1}{C} = \frac{3}{2} \times 2 = 3\Omega$ #

Prove that:

$$AD - BC = 1$$

For passive reciprocal network

Solution

- Parameters

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

- Parameters

$$V_1 = A V_2 - B I_2$$
$$I_1 = C V_2 - D I_2$$

since

$$Z_{11} = \frac{A}{C} \Rightarrow \therefore A = \frac{Z_{11}}{Z_{21}} \rightarrow [3]$$

$$Z_{12} = \frac{AD}{C} - B \Rightarrow \therefore B = \frac{Z_{11}}{Z_{21}} \times \frac{Z_{22}}{Z_{21}} \times Z_{21} - Z_{12} \therefore B = \frac{Z_{11} Z_{22}}{Z_{21}} - Z_{12} \rightarrow [4]$$

$$Z_{21} = \frac{1}{C} \Rightarrow \therefore C = \frac{1}{Z_{21}} \rightarrow [1]$$

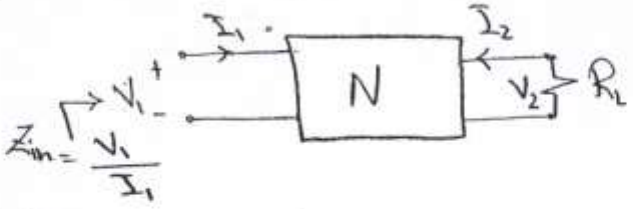
$$Z_{22} = \frac{D}{C} \Rightarrow \therefore D = \frac{Z_{22}}{Z_{21}} \rightarrow [2]$$

$$AD - BC = \frac{Z_{11}}{Z_{21}} \times \frac{Z_{22}}{Z_{21}} - \left[\frac{Z_{11} Z_{22}}{Z_{21}} - Z_{12}\right]\left[\frac{1}{Z_{21}}\right]$$

$$= \frac{Z_{11} Z_{22}}{Z_{21} Z_{21}} - \frac{Z_{11} Z_{22}}{Z_{21} Z_{21}} + \frac{Z_{12}}{Z_{21}} = \frac{Z_{12}}{Z_{21}}$$ ∴ it is reciprocal Network

$\therefore Z_{12} = Z_{21}$ $\therefore AD - BC = 1$ #

find Z_{in}, $\frac{V_2}{V_1}$



$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$V_2 = -I_2 R_L$$

$$-I_2 R_L = Z_{21} I_1 + Z_{22} I_2$$

$$-I_2 (R_L + Z_{22}) = Z_{21} I_1$$

$$\therefore I_2 = \frac{-Z_{21}}{R_L + Z_{22}} I_1$$

$$\therefore V_1 = \left[Z_{11} - \frac{Z_{21} Z_{12}}{R_L + Z_{22}} \right] I_1$$

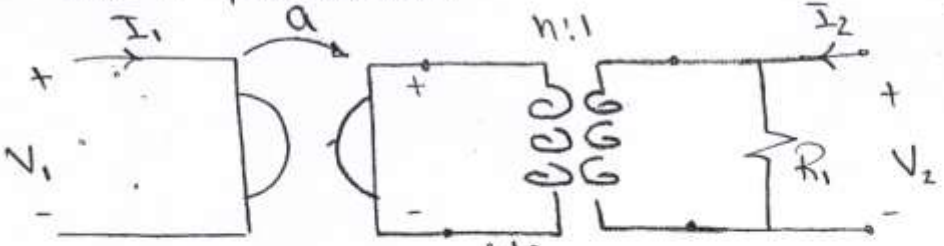
$$\therefore \boxed{Z_{in} = \frac{V_1}{I_1} = Z_{11} - \frac{Z_{21} Z_{12}}{R_L + Z_{22}}} \quad \#$$

$$\therefore V_1 = \left[Z_{11} - \frac{Z_{21} Z_{12}}{R_L + Z_{22}} \right] I_1 = \left[\frac{Z_{11} R_L + Z_{11} Z_{22} - Z_{21} Z_{12}}{R_L + Z_{22}} \right] I_1$$

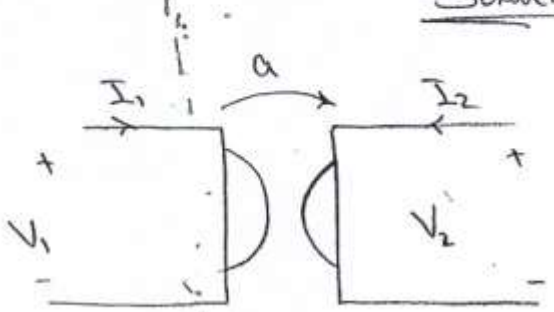
$$V_2 = \frac{+Z_{21}}{R_L + Z_{22}} I_1 \cdot R_L$$

$$\therefore \frac{V_2}{V_1} = \frac{Z_{21} R_L}{Z_{11} R_L + Z_{11} Z_{22} - Z_{21} Z_{12}}$$

Find T parameter:



Solution



Gyrator

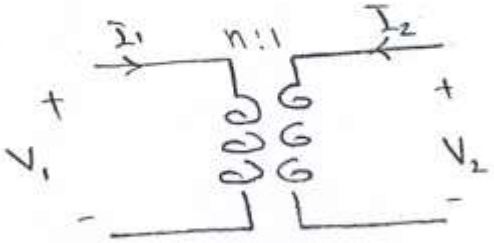
$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

$V_1 = -aI_2$, $V_2 = aI_1$

$A = 0$, $B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = a$, $C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = 1/a$, $D = 0$

$$T_1 = \begin{pmatrix} 0 & a \\ 1/a & 0 \end{pmatrix}$$



Transformer

$\therefore V_1 = nV_2$

$n = \frac{N_1}{N_2} = \frac{V_1}{V_2}$

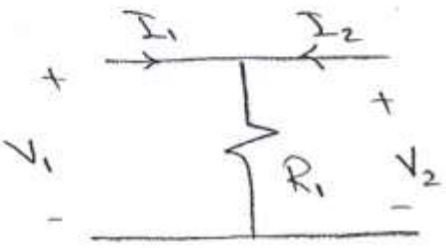
[II]

$$n = \frac{N_1}{N_2} = \frac{-I_2}{I_1}$$

$$\therefore I_1 = \frac{-1}{n} I_2$$

$$A = \frac{V_1}{V_2}\Big|_{I_2=0} = n \;,\; B = 0 \;,\; C = 0 \;,\; D = \frac{I_1}{-I_2}\Big|_{V_2=0} = \frac{1}{n}.$$

$$T_2 = \begin{pmatrix} n & 0 \\ 0 & 1/n \end{pmatrix}$$



$$V_1 = I_1 R_1 + I_2 R_1 \longrightarrow (1)$$
$$V_2 = I_2 R_1 + I_1 R_1$$
$$\therefore I_1 = \frac{1}{R_1} V_2 - I_2 \longrightarrow (2)$$

in [II]

$$V_1 = \frac{R_1}{R_1} V_2 - R_1 I_2 + R_1 I_2$$

$$\therefore V_1 = V_2$$

$$A = \frac{V_1}{V_2}\Big|_{I_2=0} = 1 \;,\; B = 0 \;,\; C = \frac{I_1}{V_2}\Big|_{I_2=0} = \frac{1}{R_1} \;,\; D = \frac{I_1}{-I_2}\Big|_{V_2=0} = 1$$

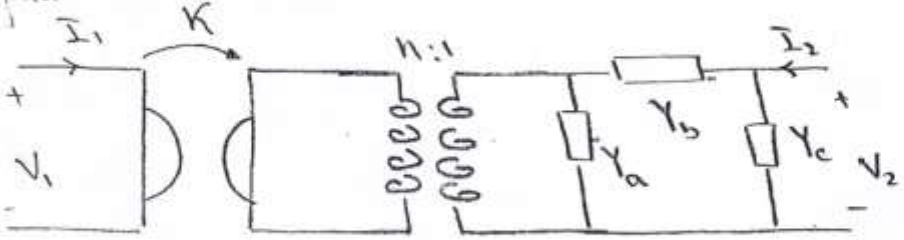
$$T_3 = \begin{pmatrix} 1 & 0 \\ 1/R_1 & 1 \end{pmatrix}$$

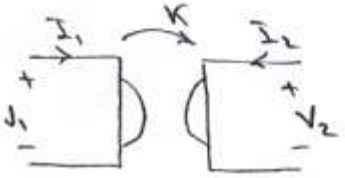
$$\overline{T} = \overline{T_1} * \overline{T_2} * \overline{T_3}$$

$$\overline{T} = \begin{pmatrix} 0 & a \\ 1/a & 0 \end{pmatrix} \begin{pmatrix} n & 0 \\ 0 & 1/n \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1/R_1 & 1 \end{pmatrix}$$

$$\overline{T} = \begin{pmatrix} 0 & a/n \\ n/a & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1/R_1 & 1 \end{pmatrix} = \begin{pmatrix} a/nR_1 & a/n \\ n/a & 0 \end{pmatrix}$$

Find T- Parameters:





$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

$$V_1 = -KI_2 \,, \quad V_2 = KI_1$$

$$A = 0 \,, \quad B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = K \,, \quad C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = 1/K \,, \quad D = 0$$

$$T_1 = \begin{pmatrix} 0 & K \\ 1/K & 0 \end{pmatrix}$$

http://www.com-dep.tk/